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AN EXPERIMENTAL INVESTIGATION OF OUTFLOW OF LIQUIDS FROM SINGLE-OUTLET VESSELS

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Abstract—This paper considers the outflow of a liquid from a single outlet vessel, i.e. a vessel in which the outflowing liquid is displaced by another fluid which enters the vessel through the same opening. The simplest possible arrangement is investigated: a sealed axisymmetric cylindrical vessel with an outlet in its base, in which water is displaced by air.

It is shown experimentally that the average liquid discharge velocity is independent of the liquid level in the vessel and the shape of the outlet for the range of outlets employed; it increases weakly with both the diameter of the vessel and the diameter of the outlet.

Key Words: single-outlet vessel, two-phase flow, draining tank, bubbles, counter-current flow, cyclic flow

1. INTRODUCTION

The work described in this paper investigates the outflow of liquids from a single-outlet vessel. In such a vessel the liquid leaving the vessel through the outlet is displaced by another fluid entering the vessel through the same opening. The simplest example of such an arrangement is an ordinary bottle, which is being emptied of its liquid content (Whalley 1987, 1991). The processes involved are familiar, but generally rather complex. They are associated with aspects of two-phase counter-current, and sometimes co-current, flow. The processes involved may not be fully understood, but one intuitively develops individual techniques for dealing with them. For example, by experimenting with different ways of bottle emptying, the discharge rate may be maximized or made less unsteady. However, various industrial applications are of somewhat greater importance. A discharge rate from a large and sealed tank, whose wall is accidentally perforated or whose isolating valve is inadvertently opened, governs the rate of the liquid loss and determines the critical times for the various corrective actions to be taken.

One asymptotic limit to this problem, when the diameter of the vessel is the same as the diameter of the outlet, is the flow of long bubbles in tubes. This has been investigated by several authors, such as Zukoski (1966), and the solutions are well known. A more general problem of tanks draining through vertical tubes has been investigated by Dougall & Kathiresan (1981) and, more recently, by Tehrani *et al.* (1992).

The work presented here deals with the simplest possible arrangement: an axisymmetric arrangement of a vertical cylindrical vessel with a central outlet in its base. The experimental work is described in the next section. This is followed by the analysis of the experimental data and discussion.

2. EXPERIMENTAL WORK

2.1. Description

The diagram of the experimental apparatus is shown in figure 1. The working vessel was a perspex cylinder, with a sealed top and an outlet in the centre of its base. The influence of the following parameters was investigated: (i) the internal vessel diameter, D_v, of 190, 348 and 600 mm, which are referred to below as the small, middle and large vessel, respectively, (ii) the vessel height, l, of 250, 500 and 996 mm, (iii) the outlet diameter, D, of 25, 40, 55 and 70 mm and (iv) the shape of the outlet, which was either cylindrical or profiled. The outlets were 20 mm long, and in the case of the profiled outlets the radius $r = 0.25\,D$.

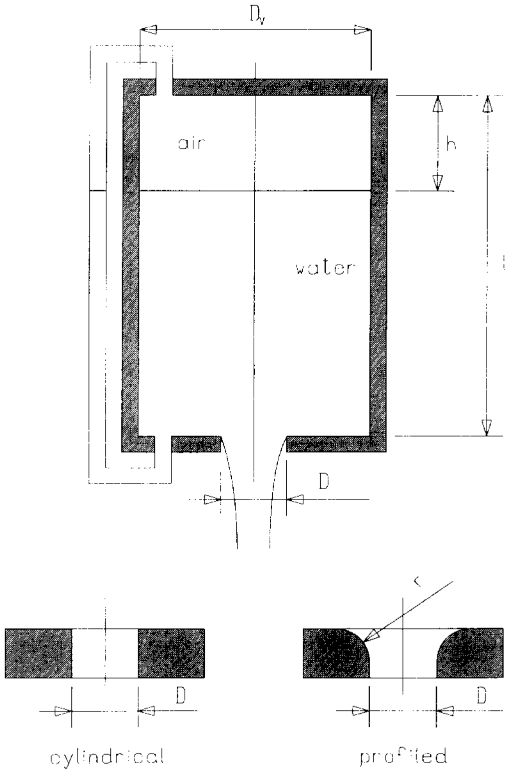

Figure 1. Diagram of the experimental apparatus.

The vessel was initially filled completely with water at room temperature and the outlet sealed with a stopper. The outlet was then opened by removing the stopper. Different ways of removing the stopper were investigated but no measurable differences in the discharge rates and other processes were observed. The opening of the outlet allowed the discharge of water from the vessel and the ingress of air into the vessel. During the discharge the air–water interface in the vessel was fluctuating, unsteady and ill-defined. This became more pronounced with increasing D/D_v. In order to smooth the variation in the position of the interface for measuring purposes, the height of the interface was measured with an 8 mm diameter sight glass. The diameter of the sight glass was chosen by a compromise. In order to minimize the fluctuation the diameter should be small, but in order to minimize the lag in the sight glass level the diameter should be large. The 8 mm diameter sight glass did not introduce any measurable lag. This then allowed the determination of the height of the air space above the interface, h, as a function of time, t. This was determined with a stopwatch by measuring the split-times for the required heights. Even with the use of the sight glass, h fluctuated from run to run and hence an average value of h over five runs was always determined. The maximum fluctuation in h such obtained was about 2 mm.

2.2. *Observations*

The outflow is a complex process. In order to understand the processes involved, photographs and video recordings were taken. These helped in the identification of the phenomena involved, but, due to the interaction of the gas, liquid and experimental apparatus, are not of sufficiently high quality to allow meaningful presentation of good photographs in this paper. Nonetheless, the following observations can be made.

The outflow of the liquid from the vessel can be described as a cyclic process. Assume that each cycle starts with the pressure in the gas space above the interface, p_G being at about atmospheric. The liquid will discharge through the outlet with a certain instantaneous discharge velocity, v, which depends, among other parameters, on the hydrostatic head, l-h, and the pressure p_G. As the liquid discharges from the vessel the pressure p_G decreases, hence decreasing the liquid discharge velocity v. As the liquid discharge velocity decreases an air bubble starts forming at the outlet. Eventually the

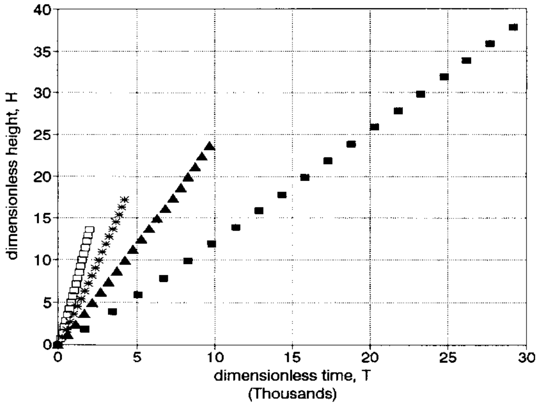

Figure 2. A plot of H vs T for $D_v = 600$ mm, $l = 996$ mm and four cylindrical outlets: $D = 25$ mm (■), $D = 40$ mm (▲), $D = 55$ mm (*), $D = 70$ mm (□).

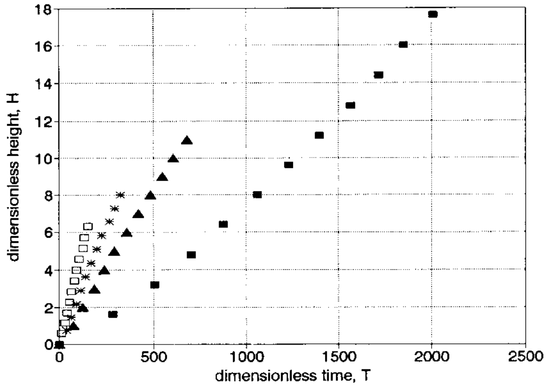

Figure 3. A plot of H vs T for $D_v = 190$ mm, $l = 500$ mm and four cylindrical outlets: $D = 25$ mm (■), $D = 40$ mm (▲), $D = 55$ mm (*), $D = 70$ mm (□).

liquid discharge decreases to such an extent that the air enters the vessel as a continuous stream of bubbles. This leads to some recovery in pressure p_G and an increase in the liquid discharge, which eventually sweeps out the incoming air, leading to the discharge of the liquid only. The process is then repeated, with a period, which depends primarily on the size of the vessel, and the size and the shape of the outlet. For each experimental arrangement the period generally increased with increasing h.

It should be noted that this flow pattern was only obtained when the air–water interface was a distance of about three outlet diameters from both the top and the bottom of the vessel. When the interface was too close to the top of the vessel, the volume of the air space was too small to allow the development of the cyclic process described above. When the interface was too close to the bottom of the vessel, the outlet became self-venting and the vessel stopped behaving as a single-outlet vessel.

2.3. *Experimental results*

Some of the experimental results are plotted in dimensionless form as H versus T, where H is defined as h/D and T is defined as $t(g/D)^{0.5}$, in figures 2 and 3. Figure 2 presents H versus T for the large vessel and the cylindrical outlet and figure 3 for the small vessel and the cylindrical outlet.

The influence of the vessel height, l, was investigated for the small vessel with l of 250 and 500 mm. It was observed that the height of the vessel in the range investigated had no measurable influence on the variation of H versus T.

3. ANALYSIS OF RESULTS

3.1. *Dimensional analysis*

The purpose of the dimensional analysis is to determine the basic governing parameters of the phenomena involved in the complex process investigated in this paper. We are not attempting to develop a general approach. Since in this investigation we are dealing just with air and water we will not consider the gas density ρ_G nor the gas viscosity. Furthermore, we will assume that the vessel and the outlets are large in the sense that viscous and surface tension forces are negligible in comparison with inertia forces. Hence we will neglect and not consider the influence of the liquid density ρ_L, the liquid viscosity and the surface tension. Since, as pointed out above, the vessel height l has no measurable influence on the variation of h with t, it is also neglected from this analysis.

The remaining variables which are considered are the height of the air space h, time t, the internal vessel diameter D_v, the outlet diameter D, gravitational acceleration g and the shape of the outlet, characterized by the discharge coefficient c_D. Hence for the present experimental arrangement the following equation can be derived

$$H = \text{function of}\left(T, \frac{D_v}{D}, c_D\right) \tag{1}$$

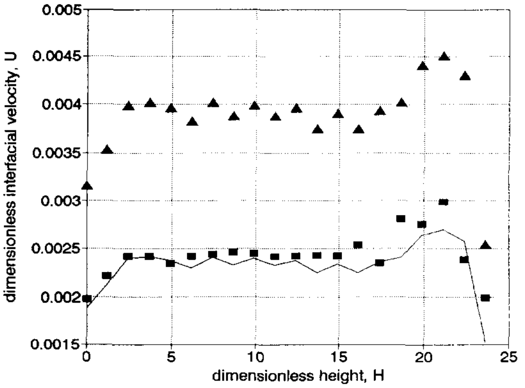

Figure 4. A plot of U vs H for $D_v = 600$ mm, $l = 996$ mm and $D = 40$ mm: cylindrical outlet (■), profiled outlet (▲) and 60% of profiled outlet (solid line).

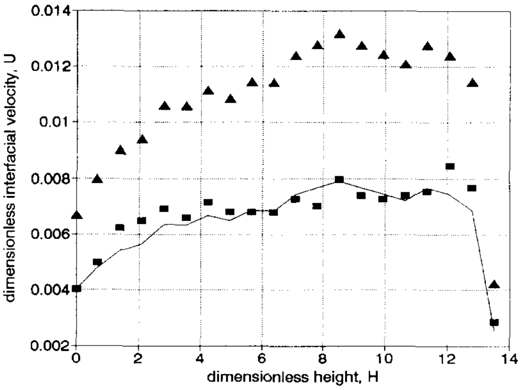

Figure 5. A plot of U vs H for $D_v = 600$ mm, $l = 996$ mm and $D = 70$ mm: cylindrical outlet (■), profiled outlet (▲) and 60% of profiled outlet (solid line).

3.2. *Gas–liquid interface*

Figures 2 and 3 give the position of the gas–liquid interface in the vessel as a function of time. These figures indicate that the dimensionless interfacial velocity, defined as $U = \Delta H/\Delta T$ and based on forward differences, is approximately invariant with H. Typical variations of the interfacial velocity versus H are given in figures 4 and 5. These figures confirm that for H in the mid-range (more than about 3 from both the top and the bottom of the vessel), the dimensionless interfacial velocity is indeed only a very weak function of the liquid level in the vessel.

Hence, it follows that

$$U = \text{function of}\left(\frac{D_v}{D}, c_D\right) \qquad [2]$$

Equation [2] indicates that the in the mid-range the interfacial velocity does not depend on any vertical dimension. This may appear counterintuitive, considering that the process is driven by gravitational forces. However, as indicated elsewhere in this paper, the flow processes depend both on gravity and on the pressure in the gas space, and the interaction between the gravitational and pressure forces is responsible for this result.

Furthermore, and as demonstrated in figures 4 and 5, the dimensionless interfacial velocities for the cylindrical outlets are on average approximately 60% of the corresponding interfacial velocities for the profiled outlets. This ratio corresponds to the ratio of the discharge coefficients, c_D, for the cylindrical and profiled outlets. The discharge coefficients were determined experimentally as a part of the present experimental work, with c_D approximately 0.6 for the cylindrical outlets and 1.0 for the profiled outlets. This implies that

$$\frac{U}{c_D} = \text{function of}\left(\frac{D_v}{D}\right) \qquad [3]$$

Finally, figures 4 and 5 also indicate that the ratio of the dimensionless interfacial velocities for the two outlets is approximately equal to the square of the ratio of the respective outlet diameters.

4. DISCUSSION

4.1. *Correlation of all experimental results*

Equation [3], which indicates the form of the dimensionless interfacial velocity when H is in mid-range, can be re-arranged as

$$\frac{\Delta H}{\Delta T} = c_D \times \text{function of}\left(\frac{D}{D_v}\right) \qquad [4]$$

and thus in the mid-range

$$H = a + bc_D \times \text{function of}\left(\frac{D}{D_v}\right) \times T \qquad [5]$$

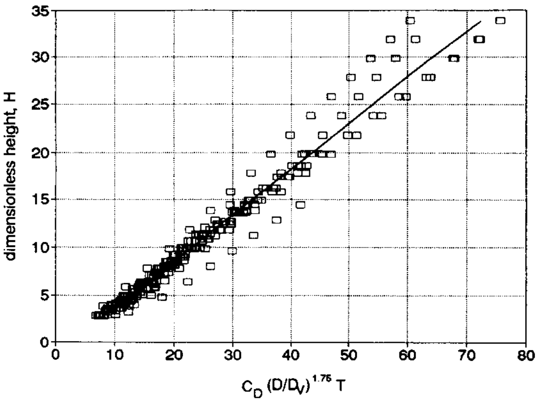

Figure 6. A plot of all mid-range experimental data of H vs $c_D\ (D/D_v)^{1.75}T$: experimental data (□) and [7] (solid line).

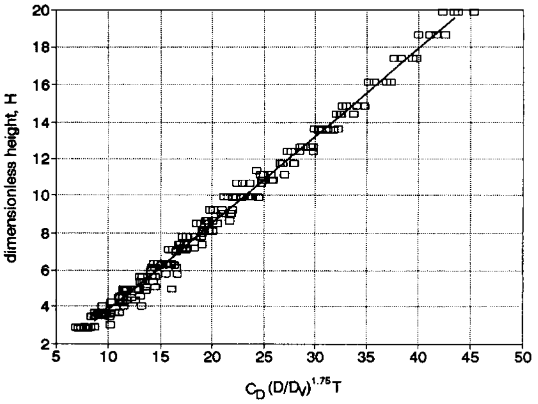

Figure 7. A plot of all mid-range experimental data, excluding the experimental data for the smallest outlet ($D = 25$ mm), of H vs $c_D\ (D/D_v)^{1.75}T$: experimental data (□) and [7] (solid line).

where a and b are constants, and they and the form of the function were determined experimentally.

It was found that the best form of the equation is given by

$$\text{function of}\left(\frac{D}{D_v}\right) = \left(\frac{D}{D_v}\right)^{1.75} \qquad [6]$$

The mid-range experimental data are plotted in figures 6 and 7; figure 6 contains all the experimental data, but figure 7 excludes the experimental data for the smallest outlet with $D = 25$ mm. Both figures indicate similar values of constants a and b, and demonstrate that the experimental data for the outlet with the smallest diameter do not correlate as well as the experimental data for the larger outlets. One of the reasons may be that for the smallest diameter outlet the liquid properties, such as viscosity and surface tension, become important. The present work is inconclusive in this area. Nevertheless, in the mid-range, i.e. for $H > 3$, the level in the vessel can be related to time by the following approximate equation

$$H = -1 + 0.47c_D\left(\frac{D}{D_v}\right)^{1.75} T \qquad [7]$$

For $H < 3$ a straight line relationship between the origin and the appropriate point at $H = 3$ can be used

$$H = 0.35c_D\left(\frac{D}{D_v}\right)^{1.75} T \qquad [8]$$

4.2. *Average liquid discharge velocity*

The rate of increase in H (or the drop in the air–water interface, as measured by the interfacial velocity U) is related to the average liquid outflow through the outlet, which is given by the product of the average liquid discharge velocity through the outlet, u, and the effective area of the outlet $c_D A_O$, where A_O is the cross-sectional area of the outlet. It should be pointed out that the interfacial velocity U is related directly to the average rate of liquid discharge through the outlet. The concept of the average liquid discharge velocity through the outlet is introduced to characterize the flow behaviour in the outlet, so that the influence of the outlet can be determined more directly. Furthermore, the outlet discharge velocities are used more commonly than interfacial velocities in other applications, and direct comparisons can thus be made.

It can be shown from the continuity equation that the average liquid discharge velocity through the outlet, u, is related to the dimensionless interfacial velocity, U, by

$$\frac{U}{c_D A} = \frac{u}{(gD)^{0.5}} \qquad [9]$$

where A is defined as A_O/A_v, A_v is the cross-sectional area of the vessel and the group $u/(gD)^{0.5}$ is the liquid discharge Froude number, Fr.

As shown above, the experimentally determined value of the constant b is approximately 0.47. Hence, using the results above, it can be shown that in the mid-range the liquid discharge Froude number, Fr, is given by

$$\mathrm{Fr} = 0.47\left(\frac{D_v}{D}\right)^{0.25} \qquad [10]$$

Equation [10] can be re-arranged to show that

$$u = 0.47g^{0.5}(D_v D)^{0.25} \qquad [11]$$

which demonstrates that in the mid-range the average liquid discharge velocity increases weakly with both the diameter of the outlet and the diameter of the vessel. In other words, for a given arrangement of vessel and outlet the average liquid discharge velocity in the mid-range is constant and thus independent of the liquid level in the vessel.

4.3. Concluding remarks

An experimental investigation of the outflow of water from single outlet vessels has been undertaken. The investigation concentrated on one of the simplest arrangements: an axisymmetric vertical cylindrical vessel with an outlet in its base. It has been shown that for such an arrangement the average liquid discharge velocity is independent of both the liquid level in the vessel and the shape of the outlets examined, and that it increases weakly with both the diameter of the outlet and the diameter of the vessel. This means that the liquid discharge Froude number depends weakly on both the diameter of the vessel and the diameter of the outlet; it increases with the former and decreases with the latter.

Finally, a correlation has been developed which relates the liquid level in the vessel with the duration of the discharge. This can be used in practical applications to estimate the discharge of liquids from such vessels.

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